About Hydration of Mg\(^{2+}\): a Quantum DFT Study

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Abstract

The hydration of magnesium bication is studied using the DFT method BPW91 / 6-31G(d,p), which gives remarkable results for the reproduction of the infra-red spectrum of liquid water. The enthalpies and the Gibbs free energies of Mg\(^{2+}\) hydration are computed using the standard and the cluster procedures, up to eight added molecules of water. The entropic contribution to the hydration phenomenon is discussed using frequency analyses.

Keywords. Hydration, magnesium bication, quantum modelling, DFT BPW91 method

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1 INTRODUCTION

Today a great number of studies has been devoted to the hydration of the Magnesium bication (Mg$^{++}$). As pointed out by Bock et al. [1], this hard divalent cation is generally found in an hexahydrated octahedral architecture and is implicated in a structural role, when associated with proteins.

Among the divalent ions of biological interest, the hardness [2,3] of magnesium (32.55 eV) is inferior to the hardest Be$^{++}$ (67.84 eV) but superior to Ca$^{++}$ (19.52 eV), Zn$^{++}$ (10.88 eV), Cu$^{++}$ (8.27 eV) and Fe$^{++}$ (7.24 eV).

The most recent theoretical studies for understanding the cation-water solvent interactions at the discrete molecular level, of the first and second shells of hydration, involve essentially the ab initio Hartree Fock, Density Functional Theory (DFT) and reaction field methods [1,4,5,6,7].

The water exchange reactions between the first and the second shells around cations is also of great interest (as seen in the review by Helm and Merbach [8] and Hartmann et al. [5] in the case of Zn$^{++}$).

As remarked by Markham et al., the cluster approach to compute the enthalpy $\Delta H$ and Gibbs free energy $\Delta G$ changes seems to be the best when combined with ab initio DFT methods.

When considering the reaction :

$$\text{Mg}^{++} + \text{cluster (H}_2\text{O})_n \rightarrow \text{cluster (Mg, nH}_2\text{O)}^{++}$$

the computation takes into account both the ligation energies (water-ion interactions) and the water-water interactions.

Recently, Bour [9] has demonstrated that computations using the DFT method BPW91 / 6-31G(d,p) , for a cluster of 214 water molecules, reproduce very well the Infra Red spectrum of liquid water; then the use of the PW91 exchange correlation functional is preferable, for the dispersion forces are not well treated by the standard DFT procedures.

In this work, we have studied as a first step the hydration of Mg$^{++}$ up to 8 water molecules, using the cluster approach and the DFT BPW91 at the same basis set level (6-31G(d,p)) than Bour.

2 PROCEDURES

All computations have been conducted with the GAUSSIAN 98 [10], Gauss View [11] and Hyperchem [12] programs on local PC stations, using ab initio methods SCF Hartree-Fock and DFT BPW91 (Becke’s 1988 exchange functional and Perdew-Wang’s 1991 gradient-corrected correlation functional).

The different clusters were optimized using firstly RHF / 6-31G* computations, then the OPT+FREQ procedure with the DFT Restricted BPW91 method at the 6-31G(d,p) basis set level.
This procedure allowed to obtain the thermochemical properties of the water clusters and Mg++ hydrated clusters.

By this way, the thermal correction to electronic energy ($E_{el}$) of a molecule (in hartree/particle) is [13,14,15]:

$$TCE = E_{trans} + E_{rot} + E_{vibr}(T)$$

(2)

Where $trans.$, $rot.$ and $vibr.$ indicate respectively the translational, rotational and vibrational motions,

with $E_{trans} = (3/2) k T$ ; $E_{rot} = (3/2) k T$ (nonlinear molecule)

and $E_{vibr}(T) = (1/2) k \sum \Theta_{Vi} + \sum \{ k \Theta_{Vi} / [ \exp (\Theta_{Vi} / T) – 1] \}$

with $i = 1, 2, ... m$

where :

- $m$ is the number of normal modes of harmonic vibrations ($m = 3n - 6$ ; $n$ number of atoms)

- $\Theta_{Vi}$ are the vibrational temperatures, related to the harmonic vibrational

wave number $\omega_i (\text{cm}^{-1})$ by : $\Theta_{Vi} = (h c / k) \omega_i$ .

In this harmonic oscillator treatment, the zero-point vibrational energy is :

$$ZPE = (1/2) k \sum \Theta_{Vi}$$

(3)

Then, the resulting thermodynamic relations are (in hartree/particle):

- thermal correction to enthalpy :
  $$TCH = TCE + k T$$

(4)

- thermal correction to Gibbs free energy :
  $$TCG = TCH – T S$$

(5)

where $S$, the computed entropy from the canonical partition functions, involves the three contributions :

$$S = S_{trans} + S_{rot} + S_{vibr}(T)$$

It is to be noted also that, in our studied supermolecules, there are not internal rotation modes [16].

And in the ideal gas approximation, we obtain :

- the total internal energy :
  $$E = E_{el} + TCE$$

(6)

- the enthalpy :
  $$H = E_{el} + TCH$$

(7)

(from $H = E + P V$)

- the Gibbs free energy :
  $$G = E_{el} + TCG$$

(8)

(from $G = H – T S$)
The standard procedures for computing the energies of hydration are based on the reaction:

\[
\text{Mg}^{++} + n \text{H}_2\text{O} \rightarrow (\text{Mg}, n \text{H}_2\text{O})^{++} + \Delta H_1 \text{ (or } \Delta G_1) \tag{9}
\]

In the cluster approach, a cluster of water containing \(n\) \(\text{H}_2\text{O}\) is symbolized by \((\text{H}_2\text{O})_n\), and a cluster of hydrated \(\text{Mg}^{++}\) containing \(n\) \(\text{H}_2\text{O}\) is symbolized by \((\text{Mg}, n\text{H}_2\text{O})^{++}\). Taking into account that \(\text{Mg}^{++}\) is hexahydrated, the structure of this cluster is \([\text{Mg}, m\text{H}_2\text{O}]\text{pH}_2\text{O}]^{++}\) with \(m\) the number of the water molecules in the first hydration shell \((m = 1..6)\), \(p\) the number of water molecules in the second hydration shell, and \(m+p=n\). In our modeling, \(p=1,2\).

Then, the hydration reactions of \(\text{Mg}^{++}\) in the cluster procedure are:

\[
\text{Mg}^{++} + (\text{H}_2\text{O})_n \rightarrow (\text{Mg}, n\text{H}_2\text{O})^{++} + \Delta H_2 \text{ (or } \Delta G_2) \tag{10}
\]

Reaction of successive binding water enthalpies or Gibbs free energy:

\[
[M\text{g}, (n-1)\text{H}_2\text{O}]^{++} + \text{H}_2\text{O} \rightarrow (\text{Mg}, n\text{H}_2\text{O})^{++} + \Delta H_3 \text{ (or } \Delta G_3) \tag{11}
\]

The resulting enthalpies (or Gibbs free energies) for (4), (5) et (6) are:

\[
\Delta H_1 = H (\text{Mg}, n \text{H}_2\text{O})^{++} - H (\text{Mg}^{++}) - n H (\text{H}_2\text{O}) \text{ } \text{ ) for the standard procedure}
\]

\[
\Delta H_2 = H (\text{Mg}, n \text{H}_2\text{O})^{++} - H (\text{Mg}^{++}) - H (\text{H}_2\text{O})_n \text{ ) for the cluster}
\]

\[
\Delta H_3 = H (\text{Mg}, n \text{H}_2\text{O})^{++} - H (\text{H}_2\text{O}) - H ([\text{Mg}, (n-1) \text{H}_2\text{O}]^{++}) \text{ ) approach}
\]

with the same procedure for obtaining \(\Delta G_1\), \(\Delta G_2\), \(\Delta G_3\).

**3 RESULTS AND DISCUSSION**

Our calculated optimized structures of water clusters correspond to local minima (with no symmetry constraint) controlled by frequency analyses.

The obtained n-mers are:

- a dimer with a structure close to those found by Xantheas and Dunning [17] with a distance between the two oxygens \(R(\text{O-O}) = 2.89\) Å and a bend hydrogen bond \(\delta_{\text{O-H-O}} = 163.7^\circ\).
- a cyclic trimer close to a perfect equilateral triangle with \(R(\text{O-O}) = 2.74\) Å (corresponding to 2 hydrogen bonded water molecules) and 3 bended-hydrogen bonds with a \(\delta\) angle which lies between 153.4° and 154.6°.
- a cyclic tetramer close to a square defined by \(R(\text{O-O}) = 2.68\) Å and 4 bended hydrogen bonds with \(\delta = 170.2^\circ\). This structure is remarkable from the electrostatic point of view because of the zero value of its dipole moment (see table 1).

These trimer and tetramer structures are close to those described in ref [17] and [18].
- Our (H2O)5 and (H2O)6 differ from the cyclic pentamer and the S6 cyclic structure defined by Xantheas and Dunning [17] and the (H2O)8 differs from the octamers with S4 and D2d structures described by Tsai and Jordan [19].

For the (H2O)5 cluster we obtain a R(O-O) varying from 2.64 to 2.76 Å. For the (H2O)6 cluster the R(O-O) distances vary from 2.69 to 2.88 Å. For the (H2O)7 cluster we obtain a variation from 2.60 to 2.85 Å. And finally for the (H2O)8 cluster we observe that the distance R(O-O) varies from 2.59 to 2.84 Å.

The thermochemistry for these water clusters is reported in table 1. We can note that the binding enthalpy for the water dimer $\Delta H_1 = \Delta H_2 = -5.66$ kcal mol$^{-1}$ is comparable to the MP4 SDQ-(FC) / 6-31G* value (-5.0 kcal mol$^{-1}$) given in ref. [4], but superior to the experimental value (-3.6 +/- 0.5 kcal mol$^{-1}$). This discrepancy is due to the level of used basis set, which not includes diffuse functions.

The obtained magnesium-water clusters, which correspond to local minima (without any imaginary frequency) are represented in figure 1. The structures of dihydrated, tetrahydrated and hexahydrated magnesium are equivalent to those computed by Xantheas and Dunning [17].

Concerning the first coordination shell, our computed distances Mg-O are close but lightly longest than those obtained by Pavlov et al. [6] who have used the B3LYP DFT method with double $\zeta$ basis sets. We obtain for the different clusters: monohydrated 1.95 Å, dihydrated 1.97 Å, trihydrated 1.99 Å, quadrihydrated 2.03 Å, quintahydrated 2.06 – 2.11 Å, hexahydrated 2.12 Å.

<table>
<thead>
<tr>
<th>NUMBER OF WATER MOLECULES</th>
<th>H (Hartree)</th>
<th>G (Hartree)</th>
<th>$H_{el}$ (Hartree)</th>
<th>S (cal mol$^{-1}$ K)</th>
<th>$\mu$ (D)</th>
<th>$\Delta H_1$ (cal mol$^{-1}$)</th>
<th>$\Delta H_2$ (cal mol$^{-1}$)</th>
<th>$\Delta G_1$ (cal mol$^{-1}$)</th>
<th>$\Delta G_2$ (cal mol$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 H2O</td>
<td>-76.386667</td>
<td>-76.48126</td>
<td>-76.41134032</td>
<td>45.164</td>
<td>2.02601</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td>2 H2O</td>
<td>-152.783555</td>
<td>-152.831635</td>
<td>-152.83347807</td>
<td>65.838</td>
<td>1.81011</td>
<td>-5.65</td>
<td>-5.66</td>
<td>-1.64</td>
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<tr>
<td>4 H2O</td>
<td>-305.601774</td>
<td>-305.643768</td>
<td>-305.70815947</td>
<td>87.541</td>
<td>0.00011</td>
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<td>-1.65</td>
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<td>5 H2O</td>
<td>-381.599163</td>
<td>-382.041212</td>
<td>-382.13497719</td>
<td>111.424</td>
<td>4.0842</td>
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<td>7 H2O</td>
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<td>-14.31</td>
<td>-5.04</td>
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<td>8 H2O</td>
<td>-611.299404</td>
<td>-611.296669</td>
<td>-611.4022996</td>
<td>140.038</td>
<td>3.3759</td>
<td>-72.17</td>
<td>-10.07</td>
<td>-7.19</td>
<td>-2.15</td>
</tr>
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</table>

The thermochemistry for these water clusters is reported in table 1. We can note that the binding enthalpy for the water dimer $\Delta H_1 = \Delta H_2 = -5.66$ kcal mol$^{-1}$ is comparable to the MP4 SDQ-(FC) / 6-31G* value (-5.0 kcal mol$^{-1}$) given in ref. [4], but superior to the experimental value (-3.6 +/- 0.5 kcal mol$^{-1}$). This discrepancy is due to the level of used basis set, which not includes diffuse functions.

The obtained magnesium-water clusters, which correspond to local minima (without any imaginary frequency) are represented in figure 1. The structures of dihydrated, tetrahydrated and hexahydrated magnesium are equivalent to those computed by Xantheas and Dunning [17].

Concerning the first coordination shell, our computed distances Mg-O are close but lightly longest than those obtained by Pavlov et al. [6] who have used the B3LYP DFT method with double $\zeta$ basis sets. We obtain for the different clusters: monohydrated 1.95 Å, dihydrated 1.97 Å, trihydrated 1.99 Å, quadrihydrated 2.03 Å, quintahydrated 2.06 – 2.11 Å, hexahydrated 2.12 Å.
Figure 1. Structures of optimized Mg-Water clusters $[\text{Mg},n\text{H}_2\text{O}]^{++}$

(a) $n = 1$ to (h) $n = 8$

Bond lengths are in angstroms and angles in degrees.
The Mulliken net charge of Magnesium is noted into brackets.
The second shell of hydration is initiated with the structures \([(Mg, 6H2O) H2O]^{++}\) and \([(Mg, 6H2O) 2 H2O]^{++}\).

We can observe on figures 1(g) and 1(h) that the water molecules belonging to the second shell are hydrogen bounded to two water molecules of the first shell. This structure was obtained also by Markham et al [4] for the \([(Mg, 6H2O) 2H2O]^{++}\) cluster.

Figure 2. Molecular geometry of the optimized \([(Mg,6H2O)3H2O]^{++}\) cluster
Bond lengths are in angstroms and angles in degrees.
The Mulliken net charge of Magnesium is noted into brackets.

Figure 2 represents the structure of the cluster with 3H2O in the second shell. We observe also in this case the presence of double hydrogen bonds to three water molecules of the first shell. It is to be noted that the distances between the oxygen atoms of the water molecules belonging to the first shell and the oxygen atoms of water molecules of the second shell are \(\sim 2.75\text{Å}\), in agreement with those reported by Pavlov et al [6] and very closed to the experimental value (2.76 Å).

Figures 1 and 2 show also the evolution of the Mulliken net charge on magnesium cation. At our basis set level, we observe a continuous decrease of the charge with the number of water molecules around the magnesium; in the case of the hexahydrated structure (figure 1f) about half of the bicationic charge was transferred to water molecules. This type of magnesium charge variation was also observed by Pavlov et al [6] in their B3LYP / 6-311+G(2d,2p) computations.
The thermochemistry of our n-hydrated divalent magnesium structures is reported in Table 2. We have reported there the energies of magnesium hydration obtained with the standard procedure ($\Delta H_1$ and $\Delta G_1$) and with the cluster procedure ($\Delta H_2$ and $\Delta G_2$). $\Delta H_3$ and $\Delta G_3$ are the values for the successive binding water enthalpies or Gibbs free energies, in a cluster procedure. The first line (0 H2O) corresponds to the Mg$^{++}$ energetic and entropic properties.

The results presented by Pavlov et al. [6] concern only the standard procedure and only internal energy variations during the magnesium hydration. Thus a direct comparison with our results is not possible. For instance, in the case of the hexahydrated magnesium, they found with our notations: $\Delta E_1 = -303.9$ kcal mol$^{-1}$ and $\Delta E_3 = -24.5$ kcal mol$^{-1}$. But our results may be compared with those obtained by Markham et al. [4] for a number of water molecules $n = 2, 4, 6, 8$, using ab initio Hartree Fock MP2 computations. The correspondence between their results and ours is $\Delta H_{298} = \Delta H_2$ and $\Delta G_{298} = \Delta G_2$. For the hexahydrated magnesium, they found $\Delta G_{298} = -266.7$ kcal mol$^{-1}$, using a MP2 (FULL) / 6-311++G** computation and $\Delta G_{298} = -286.5$ kcal mol$^{-1}$ when using the MP4 SDQ(FC) / 6-31G* procedure. Our $\Delta G_2 = -277.2$ kcal mol$^{-1}$ value is situated between these two results.

An another fact emphasized by Markham et al. [4] is to compare the $\Delta G_2$ values as $n$ increases with the experimental one: $\Delta G_{\exp} = -439.7$ kcal mol$^{-1}$. They observe that for an MP2 (FULL) / 6-311++G** computation, in the case of hexahydrated magnesium, the first shell of hydration accounts for 61% of the $\Delta G_2$ experimental value (in our case, this value is 63%). Then, when two water molecules are added in the second shell, they account for:

$$ \left[ \Delta G_2 (Mg,8H2O)^{++} - \Delta G_2 (Mg,6H2O)^{++} / \Delta G_{\exp} - \Delta G_2 (Mg,6H2O)^{++} \right] \%.$$ 

We found in our case 14.7%, a result in agreement with those obtained by Markham et al. (14%).

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**Table 2: Thermochemistry of clusters (Mg++, nH2O) (R BPW91 / 6-31G(d,p) Computations)**

<table>
<thead>
<tr>
<th>Number of Water Molecules</th>
<th>H (Eh)</th>
<th>G (Eh)</th>
<th>Eel (Eh)</th>
<th>S (cal mol$^{-1}$ K$^{-1}$)</th>
<th>$\Delta H_1$ (kcal mol$^{-1}$)</th>
<th>$\Delta H_2$ (kcal mol$^{-1}$)</th>
<th>$\Delta H_3$ (kcal mol$^{-1}$)</th>
<th>$\Delta G_1$ (kcal mol$^{-1}$)</th>
<th>$\Delta G_2$ (kcal mol$^{-1}$)</th>
<th>$\Delta G_3$ (kcal mol$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 H2O</td>
<td>-199.22106</td>
<td>-199.223058</td>
<td>-199.223058</td>
<td>35.45</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
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<td>--</td>
</tr>
<tr>
<td>2 H2O</td>
<td>-352.24641</td>
<td>-352.254039</td>
<td>-352.254039</td>
<td>77.22</td>
<td>-181.82</td>
<td>-156.16</td>
<td>-75.58</td>
<td>-147.33</td>
<td>-147.33</td>
<td>-67.54</td>
</tr>
<tr>
<td>3 H2O</td>
<td>-428.733156</td>
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<td>-428.766549</td>
<td>91.33</td>
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<td>-60.92</td>
<td>-198.99</td>
<td>-197.42</td>
<td>-51.66</td>
</tr>
</tbody>
</table>
Because the Gibbs free energy involves the entropic contribution $T \Delta S$, it is interesting to understand the entropy variation versus the $n$ added water molecules in a water cluster or around the magnesium bication. Tables 3 and 4 show the results obtained for the $(\text{H}_2\text{O})^n$ and $(\text{Mg}, \text{nH}_2\text{O})^{++}$ clusters, with the translational, rotational and vibrational contributions (with all vibrational modes taken into account) to the total entropy $S$. For all $n$, we observe that: $S[(\text{Mg},n\text{H}_2\text{O})^{++}] < S (\text{Mg}^{++}) + S(\text{(H}_2\text{O})^n)$. The variation of these entropic contributions, plotted as a function of $n$ added molecules in the clusters, is shown in figure 3. It is to be noted that our results for the total entropy values and for $n = 2, 4, 6, 8$, are comparable to those obtained by Markham et al. [4] using an ab initio calculation at the RHF / 6-31G* level; the only discrepancy concerns the hexahydrated magnesium for which these authors have found $S = 115.9$ kcal mol$^{-1}$.

Table 3 : Entropy of clusters $(\text{H}_2\text{O})^n$ (R.BPW91 / 6-31G(d,p) Computations)

$\langle T = 298.15 \text{ K} , P = 1 \text{ atm} \rangle$

<table>
<thead>
<tr>
<th>Number of Water Molecules</th>
<th>( S ) (cal mol$^{-1}$ K$^{-1}$)</th>
<th>( S_{\text{trans}} ) (cal mol$^{-1}$ K$^{-1}$)</th>
<th>( S_{\text{rot}} ) (cal mol$^{-1}$ K$^{-1}$)</th>
<th>( S_{\text{vib}} ) (cal mol$^{-1}$ K$^{-1}$)</th>
<th>Vibrational Modes</th>
<th>Stretching and Bending Vibrations</th>
<th>Intramolecular Vibrations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 H$_2$O</td>
<td>45.164</td>
<td>34.409</td>
<td>10.759</td>
<td>0.007</td>
<td>3</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>2 H$_2$O</td>
<td>68.986</td>
<td>56.673</td>
<td>21.143</td>
<td>0.042</td>
<td>12</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>3 H$_2$O</td>
<td>75.382</td>
<td>57.883</td>
<td>24.252</td>
<td>12.374</td>
<td>21</td>
<td>9</td>
<td>12</td>
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<tr>
<td>4 H$_2$O</td>
<td>78.341</td>
<td>58.741</td>
<td>26.543</td>
<td>21.055</td>
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<tr>
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<td>24</td>
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<td>6 H$_2$O</td>
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<td>29.077</td>
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<tr>
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<td>68</td>
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<tr>
<td>8 H$_2$O</td>
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<td>31.051</td>
<td>68.163</td>
<td>68</td>
<td>24</td>
<td>42</td>
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</table>

Table 4 : Entropy of clusters $(\text{Mg}^{++}, n\text{H}_2\text{O})$ (R.BPW91 / 6-31G(d,p) Computations)

$\langle T = 298.15 \text{ K} , P = 1 \text{ atm} \rangle$

<table>
<thead>
<tr>
<th>Number of Water Molecules</th>
<th>( S ) (cal mol$^{-1}$ K$^{-1}$)</th>
<th>( S_{\text{trans}} ) (cal mol$^{-1}$ K$^{-1}$)</th>
<th>( S_{\text{rot}} ) (cal mol$^{-1}$ K$^{-1}$)</th>
<th>( S_{\text{vib}} ) (cal mol$^{-1}$ K$^{-1}$)</th>
<th>Vibrational Modes</th>
<th>Stretching and Bending Vibrations</th>
<th>Intramolecular Vibrations</th>
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<tbody>
<tr>
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<td>35.462</td>
<td>35.462</td>
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<tr>
<td>1 H$_2$O</td>
<td>57.591</td>
<td>37.132</td>
<td>9.393</td>
<td>1.266</td>
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<tr>
<td>2 H$_2$O</td>
<td>77.218</td>
<td>38.196</td>
<td>22.475</td>
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<td>39.797</td>
<td>26.101</td>
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<tr>
<td>4 H$_2$O</td>
<td>103.603</td>
<td>39.577</td>
<td>27.619</td>
<td>30.397</td>
<td>33</td>
<td>12</td>
<td>21</td>
</tr>
<tr>
<td>5 H$_2$O</td>
<td>117.048</td>
<td>40.111</td>
<td>28.411</td>
<td>40.423</td>
<td>42</td>
<td>15</td>
<td>27</td>
</tr>
<tr>
<td>6 H$_2$O</td>
<td>128.228</td>
<td>40.347</td>
<td>29.089</td>
<td>58.613</td>
<td>51</td>
<td>18</td>
<td>55</td>
</tr>
<tr>
<td>7 H$_2$O</td>
<td>140.628</td>
<td>40.929</td>
<td>30.339</td>
<td>69.560</td>
<td>60</td>
<td>21</td>
<td>59</td>
</tr>
<tr>
<td>8 H$_2$O</td>
<td>149.367</td>
<td>41.496</td>
<td>31.402</td>
<td>76.920</td>
<td>69</td>
<td>24</td>
<td>45</td>
</tr>
</tbody>
</table>
Figures 3a and 3b show clearly the weight of the three contributions to the total entropy $S_{\text{tot}}$. For our water clusters, the vibrational entropy becomes a predominant contribution when $n \geq 6$ molecules. In the case of the hydrated magnesium clusters, we obtain the same result for $n \geq 5$. And as soon as $n = 3$ we observe that $S_{\text{vib}} \approx S_{\text{rot}}$ and when $n=4$ $S_{\text{vib}} \approx S_{\text{trans}}$. To explain this behaviour, it has to be reminded that $S_{\text{trans}}$ depends only on the mass of the cluster, while $S_{\text{rot}}$ depends on the 3 rotation temperatures which are related to the structure of the cluster (3 principal moments of inertia and rotational symmetry number). For all $n$, we have $S_{\text{trans}} > S_{\text{rot}}$. The vibrational contribution $S_{\text{vib}}$ depends on the vibrational modes in the clusters and in particular the most important contribution comes from the low frequency vibration modes which correspond to the intermolecular movements (libration of water molecules and translations of Mg$^{++}$ along the O-O axis of two ligated H2O) with vibrational temperatures inferior to 900K.

Figures 4a and 4b show the vibrational spectra obtained respectively for (H2O)$_6$ and (Mg, 6H2O)$_{++}$ clusters. We observe that in the case of the water cluster the IR spectrum is composed of large bands corresponding to the stretching (3102.6 to 3782.7 cm$^{-1}$) and the bending (1637.3 to 1726.6 cm$^{-1}$) of water, and in the low frequency domain to the intermolecular movements (32.6 to 1053.6 cm$^{-1}$). This spectrum is, even with only 6 molecules of water, similar to the IR spectrum of liquid water. The corresponding spectrum for the hexahydrated–magnesium is not composed of large bands but of sharp peaks: H2O stretching (~3700 and 3800 cm$^{-1}$), H2O bending (~1630 cm$^{-1}$) and librations of water molecules and translations of Mg along an O-O axis (~350, 390 and 550 cm$^{-1}$). This drastic difference between the IR spectra of water clusters and Mg-hydrated complexes is due to the strong structuration of water molecules around the bication, in the inner shell of hydration.
Figure 4. Vibrational line spectra for clusters with 6 water molecules
(a) water cluster (H2O)$_6$
(b) hydrated magnesium cluster [Mg,6H2O]$^{++}$

4 CONCLUSION

This study presents an investigation for the magnesium hydration up to 8 added molecules of water, using the DFT computations (OPT+FREQ) - RBPW91 / 6-31G(d,p).

Our results show that the cluster procedure for obtaining the enthalpies or the Gibbs free energies of hydration is preferable to the standard one. Our results are close to those obtained by Markham et al. [4] in their ab initio Hartree-Fock MP2 (FULL) 6-311++G** and MP4SDQ(FC) 6-31G* computations.

Our obtained vibrational spectra for (Mg,6H2O)$^{++}$ and (H2O)$_6$ clusters show a drastic difference in the low frequency domain which concerns the intermolecular movements. The entropy contributions may be, as remarked by Pavlov et al. [6], an important factor for stabilizing the hexahydrated form of Mg$^{++}$.

Concerning the monohydrated metallic cations, Chang [20] has recently related linear correlations between the water binding electronic energy and the $|E($HOMO-H2O$)-E($LUMO-metal Z$^{+}$)$|$ computed with DFT (BPW) and coupled cluster (CCSD(T)) methods. It seems that these methods are not very well adapted for reproducing the frontier orbitals of magnesium bication or H2O. When we observe their results, the HOMO of water is situated at $\sim -7.3$ eV; this value is far from the ionization potential of water which is 12.6 eV [2]. The same remark is valid in the case of metallic cations LUMO’s, because the ionization potential and the hardness of these species ($\eta \sim |E_{LUMO} - E_{HOMO}|/2$) are not well computed using the DFT methods. An ab initio Hartree-Fock method is better to reproduce the frontier orbitals properties.

Work is in progress to introduce the effect of diffuse functions in the used basis sets for DFT BPW91 computations, in order to get a better approach of the experimental values of the dipole moment and the polarizabilities of the water molecule, for better taking into account the dispersion forces.
5 REFERENCES


[10] Gaussian 98, Revision A.11,


